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Analytical Solution of Direct and Inverse Problems in the Internal Gravity Waves Studies by the Doppler Frequency Shift Method

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Abstract—An analytical solution of direct and inverse problems arising in the study of the internal gravity waves (IGWs) dynamic via recording of the Doppler frequency shift, is presented. The direct problem is to determine the response of the Doppler shift to IGWs in the region of the radio wave reflection point; the inverse problem is the determination of IGW parameters from data on the Doppler frequency shift. Solutions were obtained in an approximation of the isothermal ionosphere for the heights of the *F*-region. They are presented in a form convenient for their practical use and can have a wide range of applications, including the detection of soliton-like wave structures in the *F*-region of the ionosphere.

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1. INTRODUCTION

During experimental studies of the ionospheric dynamic processes by various sounding methods, one of the main conceptual problems is an adequate interpretation of the variations in the detected signal. In particular, since the signal in most cases reflects fluctuations in the electron concentration, which are associated with the dynamics of ionospheric disturbances (travelling ionospheric disturbances, TIDs), the source of the generation of such TIDs must be determined, and the role of that source can be taken on by internal gravity waves (IGWs) (Bryunelli and Namgaladze, 1988; Hocke and Schlegel, 1996) and acoustic gravitational waves (AGWs) (Grigor'ev, 1999; Nagorskii, 1999) in the neutral component, which, in turn, are excited by impulse type sources of different natures (Belashov, 1989, 1990; Pertsev and Shalimov, 1996; Drobzheva and Krasnov, 2003; see also Belashov and Vladimirov, 2005). For example, theoretical studies (Belashov, 1992, 2006; Belashov et al., 2006; Belashova et al., 2007) first predicted phenomena such as the generation of two-dimensional IGWs solitons in the regions of sharp gradients of the main ionospheric parameters (on the fronts of solar terminator and solar eclipse spot). These received qualitative confirmation in the experiments on ionospheric sounding (Belashov and Poddelsky, 1992; Galushko et al., 2007; Nasyrov et al., 2016, 2017). However, it is well known that there are no direct methods for the measurement of neutral component dynamic parameters, i.e. IGW characteristics, on ionospheric heights. Thus, the calculation of quantitative dynamic IGW characteristics from the measured data is definitely still relevant at present.

Let us consider here the problem set above on the example of one of the most effective (from the point of view of the dynamic process study method) of the ionosphere-sounding method of recording the Doppler frequency shift (DFS) of the reflected main pulse. Please note that there are two types of problems in IGW studies by the DFS method that coexist and compliment each other:

—study of the DFS response to IGWs with a priori known parameters (e.g., under a known or modeled mechanism of the IGW excitation), i.e., the direct problem, and

—interpretation of the recorded IGWs in terms of wave disturbances of the neutral component, i.e., the inverse problem.

In the current research, problems of both types are considered analytically, and their solutions are obtained in a form that is convenient for practical use. Despite of the work in which the problem of DFS reconstruction of the neutral plasma component velocity field was raised for the first time (Savel'ev, 1987), we will not introduce here restrictions on the spatial dimensions of the disturbances and consider the DFS variations for IGWs moving at near-to-horizontal angles.

2. DIRECT AND INVERSE PROBLEMS

In the isotropic case, disturbance of the radio wave phase $\delta\varphi = -(\omega/c)\int_L \delta n dl$ leads to a Doppler frequency shift

$$\omega_D = -\frac{\omega}{c}\int_L \frac{dn}{dN}\frac{\partial}{\partial t}\delta N dl, \qquad (1)$$